Int J Theor Phys (2009) 48: 1369–1378
DOI 10.1007/s10773-008-9908-6

Energy Conditions, Cosmological Solutions and Cosmic No-Hair Conjecture in Gauss-Bonnet Theory

Subenoy Chakraborty · Tanwi Bandyopadhyay

Received: 19 September 2008 / Accepted: 18 November 2008 / Published online: 27 November 2008

Abstract A detailed investigation of Gauss-Bonnet theory has been done from a different perspective. At first, the standard energy conditions are discussed and modified forms have been presented. Then some cosmological solutions have been obtained in 5D for perfect fluid assuming that the extra dimensional metric coefficient decreases with time. For some particular choice of the parameters, exponential solutions are obtained and finally, Cosmic No-Hair Conjecture has been proved for Gauss-Bonnet dilatonic scalar coupled to Einstein gravity with coupling parameter growing linearly in time.

Keywords Gauss-Bonnet theory · Cosmic No-Hair Conjecture

1 Introduction

The possibility of space-time dimensions of more than four is a standard assumption in high energy physics, particularly after the recent developments in the string theory. The idea of extra dimension was started long ago by Kaluza-Klein [1, 2] and it is generally speculated that the hidden extra dimensions may play a role in the dynamics of our usual four dimensional space-time (for review see, Applequist et al. [3, 4]). However, the recently developed idea of brane world scenario, consistent with string theory suggests our universe as a brane embedded into a five dimensional space-time, having the extra fifth dimension to be of infinite length. Thus, motivated from the string theory, a natural attempt is to consider a consistent theory of gravity in any dimension with a more general action. The effect of string theory on classical gravitational physics [5–8] is usually investigated by means of a low energy effective action, containing in addition to the Einstein-Hilbert action, squares and higher powers of curvature terms. So in general, the field equations are fourth order and bring in ghosts. However, Lovelock [9] showed that, if the higher powered curvature terms appear in a particular combination, then the field equations remain second ordered and consequently, ghosts disappear [10, 11].

S. Chakraborty (✉) · T. Bandyopadhyay
Department of Mathematics, Jadavpur University, Kolkata 700032, India
e-mail: schakraborty@math.jdvu.ac.in

The present study deals with only the first two terms of the Lovelock gravity namely, the usual Einstein-Hilbert (EH) term and the Gauss-Bonnet (GB) terms. The inclusion of GB term is important from both physical and geometrical view point. This term naturally arises as the next leading order of the α-expansion of heterotic super string theory (α^{-1} is the string tension) and plays a fundamental role in Chern-Simons gravitational theories [12, 13]. Geometrically, the addition of EH and GB terms in the action gives the most general Lagrangian for the second order field equations in five dimension, similar to EH term in four dimension [14]. In four dimension, the GB term is purely topological in nature and hence dynamics is not affected by it. The paper is organized as follows: Sect. 2 deals with the basic equations in GB theory and the energy conditions in 5D space-time for GB gravity, while cosmological solutions are presented in Sect. 3. The Cosmic No-Hair Conjecture has been examined in Sect. 4. The paper ends with a small concluding remarks in Sect. 5.

2 Energy Conditions in GB Theory

In five dimensional space-time $(M, g_{\mu\nu})$, the action for Einstein-Gauss-Bonnet (EGB) theory can be written as

$$S = \frac{1}{2}\int_M d^5x\sqrt{-g}\,[R + \alpha L_{GB} + L_{matter}] \tag{1}$$

where $L_{GB} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet Lagrangian and L_{matter} is the Lagrangian for matter field. Here, R, $R_{\mu\nu}$ and $R_{\alpha\beta\gamma\delta}$ are respectively the Ricci scalar, Ricci tensor and Riemann tensor of M. The coupling constant α has the dimension (length)^2 and is regarded as the inverse string tension ($\alpha \geq 0$). Now, variation of the action with respect to $g_{\mu\nu}$ gives the field equations

$$G_{\mu\nu} - \alpha H_{\mu\nu} = T_{\mu\nu} \tag{2}$$

where

$$H_{\mu\nu} = 2\left[RR_{\mu\nu} - 2R_{\mu\lambda}R^{\lambda}_{\nu} - 2R^{\gamma\delta}R_{\mu\gamma\nu\delta} + R^{\alpha\beta\gamma}_{\mu}R_{\nu\alpha\beta\gamma}\right] - \frac{1}{2}L_{GB}\,\delta_{\mu\nu} \tag{3}$$

is the Lovelock tensor and $T_{\mu\nu}$ is the stress energy tensor of the matter field.

The metric ansatz of the 5D space-time is taken as

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] + b^2dy^2 \tag{4}$$

If the matter is chosen in the form of a perfect fluid, i.e.,

$$T_{\mu\nu} = \mathrm{diag}[\rho(t), p(t), p(t), p(t), p_5(t)] \tag{5}$$

with $p_5(t)$, the pressure along the fifth dimension, then the explicit form of the field equations (2) are [15]:

$$3(1-n)\frac{\dot{a}^2}{a^2} + 12\alpha n\frac{\dot{a}^4}{a^4} = \rho \tag{6}$$

$$(n-2)\frac{\ddot{a}}{a} + (n - n^2 - 1)\frac{\dot{a}^2}{a^2} + \alpha\left[4n(n+1)\frac{\dot{a}^4}{a^4} - 12n\frac{\ddot{a}\dot{a}^2}{a^3}\right] = p \tag{7}$$

and

$$-3\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)+12\alpha\,\frac{\ddot{a}\dot{a}^2}{a^3}=p_5 \tag{8}$$

and the conservation of the matter gives

$$\left\{\frac{d}{dt}(a^3\rho)+p\frac{d}{dt}(a^3)\right\}-na^2\dot{a}(\rho+p_5)=0 \tag{9}$$

or equivalently,

$$\frac{d}{dt}(a^3\rho)+\tilde{p}\frac{d}{dt}(a^3)=0 \tag{10}$$

where

$$\tilde{p}=p-\frac{n}{3}(\rho+p_5) \tag{11}$$

If the above field equations are considered as the Einstein field equations in Einstein gravity for the metric (4), then the effective density and pressures become

$$\begin{cases}\rho_{eff}=\rho-12\alpha nH^4\\ p_{eff}=p-4\alpha nH^4(n+1-3q)\\ p_{(5)eff}=p_5-12\alpha qH^4\end{cases} \tag{12}$$

where $H=\frac{\dot{a}}{a}$ and $q=-\frac{a\ddot{a}}{\dot{a}^2}$ are respectively the Hubble parameter and deceleration parameter of the usual 4D space-time. Note that, here we have assumed the extra dimension to be compactified with the expansion of the usual three spatial dimensions and we assume according to Mohammedi [16] (see also [15])

$$b(t)\sim a^{-n}(t),\quad n>0 \tag{13}$$

2.1 Energy Conditions

At first, we state the energy conditions in five dimensional space-time. Suppose the energy momentum tensor is chosen as anisotropic perfect fluid i.e.,

$$T_\mu^\nu=\text{diag}\,(\rho(t),p_1(t),p_2(t),p_3(t),p_4(t)) \tag{14}$$

Null Energy Condition It states that the condition for the convergence (geodesic focusing) of hypersurface orthogonal congruence of null geodesics defined by a null vector field K^μ is [17–19]

$$R_{\mu\nu}K^\mu K^\nu=T_{\mu\nu}K^\mu K^\nu\geq 0 \tag{15}$$

This null energy condition is in coordinate invariant form and for the above form of energy momentum tensor (14), it gives

$$\rho+p_i\geq 0\quad\forall i=1,2,3,4 \tag{16}$$

Weak Energy Condition Physically, it means that the local energy density as measured by any timelike observer is positive. Mathematically, it states that [17–19]

$$T_{\mu\nu} V^\mu V^\nu \geq 0 \tag{17}$$

for any time-like vector V^μ. Null energy condition is implied (by continuity) by the weak energy condition. Explicitly, the condition states that

$$\rho \geq 0 \quad \text{and} \quad \rho + p_j \geq 0 \quad \forall j = 1, 2, 3, 4 \tag{18}$$

Strong Energy Condition This energy condition states that any hypersurface orthogonal timelike congruence should be convergent (due to attractive nature of gravity). Then using Raychaudhuri equation, the mathematical form of it is [17–19]

$$R_{\mu\nu} V^\mu V^\nu = \left(T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu} \right) V^\mu V^\nu \geq 0 \tag{19}$$

for any timelike vector field V^μ.

The condition (19) is also coordinate invariant. For the matter field (14), the explicit form of the above inequality (19) is

$$\rho + p_j \geq 0 \quad \forall j = 1, 2, 3, 4 \quad \text{and} \quad 2\rho + \sum_{j=1}^{4} p_j \geq 0 \tag{20}$$

Note that strong energy condition implies null energy condition but it does not imply the weak energy condition in general.

Dominant Energy Condition Mathematically, it states that [17–19]

$$T_{\mu\nu} V^\mu V^\nu \geq 0 \quad \text{and} \quad T_{\mu\nu} V^\nu \ \text{ is not spacelike} \tag{21}$$

for any timelike vector V^μ. For the matter distribution (14), this energy condition demands

$$\rho \geq 0 \quad \text{and} \quad p_j \in [-\rho, +\rho], \quad \forall j = 1, 2, 3, 4 \tag{22}$$

which means that locally measured energy density is always positive and that the energy flux is timelike or null. Note that the dominant energy condition implies weak energy condition and thus also the null energy condition, but does not necessarily imply the strong energy condition.

Now for the present problem using the effective matter density and pressure (given by (12)), the above energy conditions give [20]

(a) Null energy condition:

$$\begin{aligned} &\rho + p \geq 0, \quad \rho + p_5 \geq 0 \\ &\alpha(3q - n - 4) \geq 0 \\ &\alpha(n + q) \leq 0 \end{aligned} \tag{23}$$

(b) Weak energy condition: In addition to the restrictions (23), we have

$$\rho - 12\alpha n H^4 \geq 0 \tag{24}$$

(c) Strong energy condition: In addition to the null energy conditions (23), we have

$$\rho + 3p + p_5 - 12\alpha n H^4(n+2) - 12\alpha q H^4(1-3n) \geq 0 \tag{25}$$

(d) Dominant energy condition: Here, in addition to the restrictions (23) and (24), we have the conditions

$$\begin{aligned} \rho - p + 4\alpha n H^4(n-2-3q) \geq 0 \\ \rho - p_5 + 12\alpha H^4(q-n) \geq 0 \end{aligned} \tag{26}$$

3 Cosmological Solutions

In GB theory, we have three field equations containing four unknowns. So we may assume one more relation for consistent solution. We determine the solutions for the following assumptions.

Isotropic Pressure: $p_5 = p$ Equating (7) and (8), the differential equation for the scale factor a becomes

$$\frac{\ddot{a}}{a} - (n-2)\frac{\dot{a}^2}{a^2} + 4\alpha\frac{\dot{a}^2}{a^2}\left[n\frac{\dot{a}^2}{a^2} - 3\frac{\ddot{a}}{a}\right] = 0 \tag{27}$$

which has a first integral

$$4\sqrt{\alpha}\, z \tanh^{-1}(2\sqrt{\alpha}\, z) + (n-3)(t \pm t_0)z + 1 = 0 \tag{28}$$

where $z = \frac{\dot{a}}{a}$.

In particular, for $n = 3$, one gets the explicit solution as

$$a(t) = e^{C(t+t_0)}, \quad t_0 = \text{constant}, C = \frac{1}{2\sqrt{\alpha}}\tanh\left(-\frac{1}{4\sqrt{\alpha}}\right)$$

Barotropic Equation of State: $p = \omega\rho$ Eliminating ρ between (6) and (7) using the equation of state, the differential equation for a is

$$(2-n)\frac{\ddot{a}}{a} + \left[3\omega(1-n) - (n-n^2-1)\right]\frac{\dot{a}^2}{a^2} + 12\alpha n\frac{\dot{a}^2}{a^2}\left[\frac{\ddot{a}}{a} + \frac{12\alpha n\omega - 4\alpha n(n+1)}{12\alpha n}\frac{\dot{a}^2}{a^2}\right] = 0 \tag{29}$$

which has a first integral of the form

$$uz\tan^{-1}\left(\frac{v}{z}\right) = z\left[\frac{3\omega(1-n) + (n^2-2n+3)}{(2-n)}\right](t \pm t_0) - 1 \tag{30}$$

where $z = \frac{\dot{a}}{a}$, $u = \frac{(2+3\omega-n)^{1/2}}{6\sqrt{\alpha n}(2-n)}\,\frac{[\{3\omega(1-n)+(n^2-2n+3)\}^2-12\alpha n(2-n)]}{[3\omega(1-n)+(n^2-2n+3)]^{1/2}}$ and $v = \frac{[3\omega(1-n)+(n^2-2n+3)]^{1/2}}{2\sqrt{\alpha n}(2+3\omega-n)^{1/2}}$.

Here also, explicit solutions are possible only for ($\omega = \frac{5}{3}$, $n = 1$) and $n = 2$. The respective solutions are the following

$$\begin{aligned} &a(t) = (t+t_0)^2 \\ &a(t) = \exp\left[\sqrt{\frac{\omega-1}{8\omega}}\left\{t + t_0' - \frac{8}{\omega-1}\ln\left(e^{\left[\frac{\omega-1}{4}\right]t_0} + e^{-\left[\frac{\omega-1}{4}\right]t}\right)\right\}\right], \quad t_0, t_0' = \text{constants} \end{aligned} \tag{31}$$

So for $\omega = \frac{5}{3}$, $n = 1$, we have power law solution while for $n = 2$, we have a combination (in product form) of exponential and power law forms of solution, for any choice of ω.

4 Cosmic No-Hair Conjecture in GB Theory

The Cosmic No-Hair Conjecture (CNHC) in general terminology states that "All expanding universe models with a positive Cosmological Constant asymptotically approach the de Sitter solution". Gibbons and Hawking [21] and then Hawking and Moss [22] developed this conjecture to address the question of whether the universe evolves to a homogeneous and isotropic state during an inflationary era. A formal proof of CNHC was given by Wald [23] for homogeneous Bianchi models with a Cosmological Constant. Later, Kitada and Maeda [24] proved the CNHC for Bianchi models in power-law inflation and subsequently Chakraborty et al. [25] extended this conjecture for a scalar field with ϕ^4-potential. As in inflationary models, the vacuum energy is almost a constant and behaves as a cosmological constant; so the conjecture is applicable. For power-law inflation, scalar field with exponential potential is considered and it is assumed that the potential is flat (a cosmological constant) during slow roll. The ϕ^4-potential used by Chakraborty et al. is assumed to have a constant additive part which behaves as a cosmological constant at late times. In this section, we shall examine the CNHC in GB theory.

We start with Gauss-Bonnet-Dilatonic scalar coupling with Einstein's gravity in four dimensions. The action for the above GB interaction is given by [26]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \frac{\alpha(\phi)}{8}L_{GB} - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) + L_m\right] \tag{32}$$

where $\alpha(\phi)$ is the coupling parameter and L_m is the matter Lagrangian. The potential $V(\phi)$ may be exponential (behaves as flat during slow roll) or ϕ^4-type with a constant additive term (behaves as cosmological constant at late time). For Bianchi cosmologies (except Bianchi type IX), the field equations (obtained by varying S with respect to $g_{\mu\nu}$ and dilaton field ϕ) are (in terms of Hubble parameter H)

$$2\dot{H} + 3H^2 = -\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) + 2\alpha'\dot{\phi}(H\dot{H} + H^3) + (\alpha'\ddot{\phi} + \alpha''\dot{\phi}^2)H^2 + p_m\right] \tag{33}$$

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\alpha'\dot{\phi}H^3 + \rho_m \tag{34}$$

and

$$\ddot{\phi} + 3H\dot{\phi} + V' = 3\alpha' H^2(\dot{H} + H^2) \tag{35}$$

where $\kappa(= 8\pi G) = \hbar = c = 1$, $'$ and $\cdot$ stand for differentiation with respect to ϕ and proper time t, and ρ_m, p_m are respectively the energy density and pressure corresponding to background matter distribution. This arbitrary matter distribution is assumed to satisfy the dominant and strong energy conditions namely

$$\begin{aligned} T_d[t] &= T_{\mu\nu}t^\mu t^\nu \geq 0 \\ T_s[t] &= \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)t^\mu t^\nu \geq 0 \end{aligned} \tag{36}$$

with $T_{\mu\nu}$, the energy momentum tensor and t^μ, a unit timelike vector.

Using the idea of Sanyal [26], we choose the coupling parameter such that

$$\alpha'\dot{\phi} = \alpha_0, \quad \text{a constant} \tag{37}$$

This assumption implies $\alpha(\phi(t)) = \alpha_0 t$, which grows in time to have significant contribution at later epoch of cosmological evolution and may explain recent accelerating expansion of the universe. This choice of α simplifies the above field equations to

$$2\dot{H} + 3H^2 = -\left[\frac{1}{2}\dot{\phi}^2 - V(\phi) + 2\alpha_0 H\dot{H} + 2\alpha_0 H^3 + p_m\right] \tag{38}$$

$$3H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\alpha_0 H^3 + \rho_m \tag{39}$$

and

$$(\ddot{\phi} + V')\dot{\phi} + 3H\dot{\phi}^2 = 3\alpha_0 H^2(\dot{H} + H^2) \tag{40}$$

For studying cosmological evolution, we consider the initial value constraint i.e., the Hamiltonian constraint namely,

$$G_{\mu\nu}n^\mu n^\nu - \left[\frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\alpha_0 H^3 + T_d(n)\right] = 0 \tag{41}$$

and the Raychaudhuri equation

$$R_{\alpha\beta}n^\alpha n^\beta - \left[\dot{\phi}^2 - V(\phi) + 3\alpha_0 H\dot{H} - \frac{3}{2}\alpha_0 H^3 + T_s(n)\right] = 0 \tag{42}$$

Here n^α is the unit normal to the homogeneous hypersurface Σ by $3+1$ decomposition of the four dimensional manifold. The metric and the extrinsic curvature of this 3-space Σ are defined as [25]

$$q_{ab} = g_{ab} + n_a n_b \tag{43}$$

and

$$K_{ab} = \frac{1}{2N}\frac{\partial g_{ab}}{\partial t} \tag{44}$$

Also we can write,

$$K_{ab} = \frac{1}{3}Kq_{ab} + \sigma_{ab} \tag{45}$$

where $K = K_{ab}q^{ab}$ is the trace of the extrinsic curvature and σ_{ab} is the shear of the timelike geodesic congruence orthogonal to the homogeneous hypersurfaces. Now, in terms of the 3-space variables, the dynamical equations (41) and (42) can be written as

$$K^2 = 3\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\alpha_0 H^3\right] + \frac{3}{2}\sigma_{ab}\sigma^{ab} - \frac{3}{2}{}^{(3)}R + 3T_d(n) \tag{46}$$

$$\dot{K} = \left[-\dot{\phi}^2 + V(\phi) - 3\alpha_0 H\dot{H} - \frac{3}{2}\alpha_0 H^3\right] - \frac{1}{3}K^2 - \sigma_{ab}\sigma^{ab} - T_s(n) \tag{47}$$

where ${}^{(3)}R$ is the scalar curvature of the homogeneous hypersurface Σ and can be shown to be negative (except Bianchi IX, for details see [23, 25]), i.e., ${}^{(3)}R \le 0$. From the constraint equation (46), we see that second, third and fourth terms on the RHS are positive, so we write

$$K^2 > 3\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\alpha_0 H^3\right] \tag{48}$$

Let us define [24, 25]

$$K_\phi = K^2 - 3\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\alpha_0 H^3\right] = \frac{3}{2}\sigma_{ab}\sigma^{ab} - \frac{3}{2}{}^{(3)}R + 3T_d(n) \tag{49}$$

Note that K_ϕ is positive definite and can be interpreted as the modified form of the constraint equation. Now, time differentiation of (49) (after simple algebra) gives

$$\dot{K}_\phi = -\frac{2}{3}KK_\phi - 2K(\sigma_{ab}\sigma^{ab} + T_s) \tag{50}$$

or we can write

$$\dot{K}_\phi \le -2HK_\phi \tag{51}$$

As $K_\phi \ge 0$, so integrating this inequality we get

$$0 \le K_\phi \le K_{\phi 0} e^{-2\int H dt} \tag{52}$$

We shall now consider two time epoch separately:

Inflationary Era During exponential expansion (inflation), H can be assumed to be constant (say H_0) and we have

$$0 \le K_\phi \le K_{\phi 0} e^{-2H_0 t} \tag{53}$$

This shows that K_ϕ falls off exponentially with time and tends to zero.

Power Law Expansion If instead of constant value, H varies inversely with time (i.e., $\frac{H_0}{t}$), then form the inequality (52), K_ϕ is bounded as

$$0 \le K_\phi \le K_{\phi 0} t^{-2H_0} \tag{54}$$

i.e., K_ϕ decreases with time in a power law fashion.

Thus, the behaviour of K_ϕ is not affected by the nature of the potential and the presence of the GB term. Moreover, after rearranging the Raychaudhuri equation (47), we write

$$\dot{K} + \frac{1}{3}K^2 \le V(\phi) - \frac{1}{3}\alpha_0 K\dot{K} \tag{55}$$

As during slow-roll inflation, the scalar field decreases and the potential acts as a cosmological constant (Λ), so inequality (55) can be simplified as

$$\left(1 + \frac{\alpha_0 K}{3}\right)\dot{K} \le \Lambda - \frac{1}{3}K^2 \tag{56}$$

Now integrating this inequality, we obtain (after simple algebra)

$$K \le \frac{3}{\alpha_0} - e^{-\frac{t}{\alpha_0}} \tag{57}$$

where $\alpha_0\sqrt{\frac{\Lambda}{3}}$ is chosen to be unity. Thus the expansion rate K approaches $\frac{3}{\alpha_0}$ exponentially over the time scale α_0. Also from the (49) we have

$$A \leq \frac{1}{3}\left(K^2 - \frac{9}{\alpha_0^2}\right) \leq e^{-\frac{2t}{\alpha_0}} - \frac{6}{\alpha_0}e^{-\frac{t}{\alpha_0}} \tag{58}$$

where A may be any one of $\frac{1}{2}\,\sigma_{ab}\sigma^{ab}$, $-\frac{1}{2}{}^{(3)}R$ and $T_d(n)$. Hence, the shear, the curvature of the homogeneous hypersurfaces and the matter energy density rapidly approach to zero.

Hence, as K approaches $\frac{3}{\alpha_0}$, σ_{ab} and ${}^{(3)}R$ tend to zero, so that at late times, the spatial metric has the approximate form

$$h_{ab}(t) \approx e^{\frac{2(t-t_0)}{\alpha_0}} h_{ab}(t_0) \tag{59}$$

5 Concluding Remarks

Therefore, for Bianchi cosmological models (except IX), both exponential and power law inflation are possible for a scalar field with arbitrary potential (with a constant additive term). Also after inflation ($t \gg \alpha_0$), the universe will appear to be matter free with nearly flat spatial sections (isotropized) having constant rate of isotropic expansion ($K \to \frac{3}{\alpha_0}$). The constant additive term in the potential is assumed to be related to the GB coupling parameter α_0 and influences the dynamics of the late universe.

In Sect. 2, we have obtained constraints on GB gravity theory from the so called energy conditions. We have followed the approach of Santos et al. [20] who, starting from the Raychaudhuri equation along with the requirement that gravity is attractive, derived the null and strong energy conditions in the frame work of $f(R)$ gravity and showed that the restrictions may be obtained directly from the effective energy momentum tensor in Jordan frame. In Sect. 3, we have obtained cosmological solutions for 5D GB theory. It has been shown that exponential expansion (i.e., acceleration) may be obtained from standard barotropic fluid.

Acknowledgements The authors are thankful to IUCAA for warm hospitality. Also, T.B. is thankful to CSIR, Govt. of India, for awarding SRF.

References

1. Kaluza, T.: Sitz. Preuss. Akad. Wiss. Berl. (Math. Phys.) **1921**, 966 (1921)
2. Klein, O.: Z. Phys. **37**, 895 (1926)
3. Applequist, T., Chodos, A., Frend, P.G.O.: Modern Kaluza-Klein Theories. Addison-Wesley, Reading (1987)
4. Wesson, P.S.: Five Dimensional Physics: Classical and Quantum Consequences of Kaluza-Klein Cosmology. World Scientific, Singapore (2006)
5. Candelas, P., Horowitz, G.T., Strominger, A., Witten, E.: Nucl. Phys. B **258**, 46 (1985)
6. Greens, M.B., Schwarz, J.H., Witten, E.: Superstring Theory. Cambridge University Press, Cambridge (1987)
7. Lust, D., Theusen, S.: Lectures on String Theory. Springer, Berlin (1989)
8. Polchinski, J.: String Theory. Cambridge University Press, Cambridge (1998)
9. Lovelock, D.: J. Math. Phys. **12**, 498 (1971)
10. Zwiebach, B.: Phys. Lett. B **156**, 315 (1985)
11. Zumino, B.: Phys. Rep. **137**, 109 (1986)
12. Chamseddine, A.H.: Phys. Lett. B **233**, 291 (1989)
13. Muller-Hoissen, F.: Nucl. Phys. B **349**, 235 (1990)

14. Lanczos, C.: Ann. Math. **39**, 842 (1938)
15. Andrew, K., Bolen, B., Middleton, C.A.: Gen. Relativ. Gravit. **39**, 2061 (2007)
16. Mohammedi, N.: Phys. Rev. D **65**, 104018 (2002)
17. Hawking, S., Ellis, G.F.R.: The Large Scale Structure of Space-Time. Cambridge University Press, Cambridge (1973)
18. Visser, M.S., Wormholes, L.: From Einstein to Hawking. AIP Press, New York (1995)
19. Poisson, E.: A Relativist's Toolkit (The Mathematics of Black Hole Mechanics). Cambridge University Press, Cambridge (2004)
20. Santos, J., Alcaniz, J.S., Rebouças, M.J., Carvalho, F.C.: Phys. Rev. D **76**, 083513 (2007)
21. Gibbons, G.W., Hawking, S.W.: Phys. Rev. D **15**, 2738 (1977)
22. Hawking, S.W., Moss, I.G.: Phys. Lett. B **110**, 35 (1982)
23. Wald, R.M.: Phys. Rev. D **28**, 2118 (1983)
24. Kitada, Y., Maeda, K.: Phys. Rev. D **45**, 1416 (1992)
25. Chakraborty, S., Paul, B.C.: Phys. Rev. D **64**, 127502 (2001)
26. Sanyal, A.K.: Phys. Lett. B **645**, 1 (2007)